CBCS SCHEME

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15CS36

Third Semester B.E. Degree Examination, Feb./Mar. 2022
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Let p, q be primitive statements for which the implication $p \to q$ is false. Determine the truth values for each of the following:
i) $p \wedge q$ ii) $\neg p \vee q$ iii) $q \to p$ iv) $\neg q \to \neg p$ **(04 Marks)**

b. Verify that $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a Tautology. **(04 Marks)**

c. Establish the validity of the following argument:

$$\begin{array}{l} \forall x, [p(x) \vee q(x)] \\ \exists x(\neg p(x)) \\ \forall x [\neg q(x) \vee r(x)] \\ \underline{\forall x [s(x) \to \neg r(x)]} \\ \therefore \exists x \neg s(x) \end{array}$$

(04 Marks)

d. Use method of exhaustion to show that every even integer n with $2 \le n \le 26$ can be written as a sum of at most 3 perfect squares. **(04 Marks)**

OR

2 a. For the universe of all real numbers, define the following open statements, $p(x) : x \ge 0$, $q(x) : x^2 \ge 0$, $r(x) : x^2 - 3 > 0$. Determine the truth value of the following statements.
i) $\exists x, p(x) \wedge q(x)$ ii) $\forall x, p(x) \to q(x)$ iii) $\forall x, q(x) \to r(x)$ **(04 Marks)**

b. Without using truth tables, prove the following logical equivalence
$[(p \vee q) \wedge (p \vee \neg q)] \vee q \Leftrightarrow p \vee q$. **(04 Marks)**

c. Find the negation of the following quantified statement:
$\forall x, \exists y [\{p(x, y) \wedge q(x, y)\} \to r(x, y)]$ **(04 Marks)**

d. Disprove the statement: "The sum of two odd integers is an odd integer". **(04 Marks)**

Module-2

3 a. Prove by mathematical induction that for every positive integer n, 5 divides $n^5 - n$. **(04 Marks)**

b. Find an explicit definition of the sequence defined recursively by $a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \ge 2$. **(04 Marks)**

c. Find the number of proper divisors of 44100. **(04 Marks)**

d. Find the coefficient of
i) $x^2 y^2 z^3$ is the expansion of $(3x - 2y - 4z)^7$
ii) $a^2 b^3 c^2 d^5$ is the expansion of $(a + 2b - 3c + 2d + 5)^{16}$ **(04 Marks)**

OR

4 a. Prove by mathematical induction that

$$1.2 + 2.3 + 3.4 + \ldots.. + n \cdot (n+1) = \frac{1}{3} n(n+1)(n+2)$$

(04 Marks)

b. For the Fibonacci sequence $F_0, F_1, F_2, \ldots$. Prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$ **(04 Marks)**

c. Prove the following identities:
   i) $c(n, r-1) + c(n, r) = c(n+1, r)$
   ii) $c(m, 2) + c(n, 2) = c(m+n, 2) - mn$ **(04 Marks)**

d. Find the number of non negative integer solutions of the inequality $x_1 + x_2 + \ldots + x_6 < 10$. **(04 Marks)**

Module-3

5 a. Show that every set of seven distinct integers includes two integers x and y such that at least one of x + y or x – y is divisible by 10. **(04 Marks)**

b. Let f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $(gof)\, x^2 = ax^2 - 9x + 3$. Determine a and b. **(04 Marks)**

c. Let A = {1, 2, 3, 4, 5, 6} and R be a relation on A defined by aRb if and only if a is a multiple of b. Represent the relation R as a matrix and draw its digraph. **(04 Marks)**

d. Draw the Hasse diagram representing the positive divisors of 36. **(04 Marks)**

OR

6 a. Let A and B be finite sets with $|A| = m$ and $|B| = n$
   i) Find how many functions are possible from A to B.
   ii) If there are 2187 functions from A to B and $|B| = 3$ what is $|A|$? **(05 Marks)**

b. ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that at least 2 of these points are such that the distance between them is less than 1/2cm. **(05 Marks)**

c. Let A = B = C = R and $f: A \to B$ and $g: B \to C$ be defined by $f(a) = 2a + 1$, $g(b) = \frac{1}{3}b$, $\forall a \in A, \forall b \in B$. Compute gof and show that gof is invertible. What is $(gof)^{-1}$? **(06 Marks)**

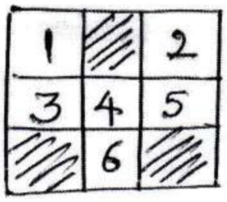
Module-4

7 a. How many integers between 1 and 300 (inclusive) are
   i) Divisible by atleast one of 5, 6, 8?
   ii) Divisible by none of 5, 6, 8? **(04 Marks)**

b. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(04 Marks)**

c. For the positive integers 1, 2, 3,….n there are 11660 derangements where 1, 2, 3, 4, 5 appear in the first five positions. What is the value of n? **(04 Marks)**

d. Find the rook polynomial for the board C shown below (made up of unshaded parts). **(04 Marks)**



OR

8 a. Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$. **(06 Marks)**

b. In how many ways can the integers 1, 2, 3,…. 10 be arranged in a line so that no even integer is in its natural place. **(05 Marks)**

c. Solve the recurrence relation
$a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \geq 2$ given that $a_0 = 5$, $a_1 = 12$. **(05 Marks)**

Module-5

9 a. Define isomorphism. Verify the two graphs are isomorphic.

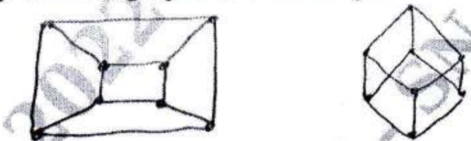


Fig.Q.9(a)

(05 Marks)

b. Let $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ be two trees. If $|E_1| = 19$ and $|V_2| = 3|V_1|$, determine $|V_1|$, $|V_2|$ and $|E_2|$. **(05 Marks)**

c. Construct an optimal prefix code for the letters of the word 'ENGINEERING'. Hence deduce the code for this word. **(06 Marks)**

OR

10 a. Show that a connected graph with exactly two vertices of odd degree has an Euler Trial. **(05 Marks)**

b. Using the merge sort method, sort the list 7, 3, 8, 4, 5, 10, 6, 2, 9. **(05 Marks)**

c. Consider the prefix code
a : 111, b : 0, c : 1100, d : 1101, e : 10
Using this code decode the following sequences:
i) 1001111101 ii) 10111100110001101 iii) 1101111110010 **(06 Marks)**

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